Nonrigid Iterative Closest Points for Registration of 3D Biomedical Surfaces

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Abstract

Advanced 3D optical and laser scanners bring new challenges to computer graphics. We present a novel nonrigid surface registration algorithm based on Iterative Closest Point (ICP) method with multiple correspondences. Our method, called the Nonrigid Iterative Closest Points (NICPs), can be applied to surfaces of arbitrary topology. It does not impose any restrictions on the deformation, e.g. rigidity or articulation. Finally, it does not require parametrization of input meshes. Our method is based on an objective function that combines distance and regularization terms. Unlike the standard ICP, the distance term is determined based on multiple two-way correspondences rather than single one-way correspondences between surfaces. A Laplacian-based regularization term is proposed to take full advantage of multiple two-way correspondences. This term regularizes the surface movement by enforcing vertices to move coherently with their 1-ring neighbors. The proposed method achieves good performances when no global pose differences or significant amount of bending exists in the models, for example, families of similar shapes, like human femur and vertebral models.

Keywords: surface registration, nonrigid iterative closest points (NICPs), multiple two-way correspondences, bone

1. Introduction

The proliferation of optical and laser scanners leads to a variety of applications such as computer-aided design, virtual reality and medical diagnosis and treatment [1]. The goal of surface registration is to find a transformation which best superposes one surface with another [2]. Typically, this is done by transforming the source surface to make it as close as possible to the target surface. There are two variants of the registration problem: rigid and nonrigid. In rigid registration, the transformation that is applied to the source surface is required to be an orientation-preserving isometry (superposition of a rotation and a translation). In nonrigid registration, one is allowed to deform the source surface to improve the solution. This paper addresses the nonrigid registration problem, which has numerous applications in medical imaging, computer graphics and computer vision. Non-rigid registration algorithms can provide correspondence information for two similar shapes by deforming one to another. The correspondence information can be used in

\begin{itemize}
  \item example-based segmentation [3];
  \item interpolating [3] or animating [4] models;
  \item computing low-dimensional representations of shape families, such as PCA models [5, 6, 7];
\end{itemize}

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• comparing different shapes in a same family (for example, analyzing the growth of an organ over time) [8, 9].

Our registration algorithm follows the Non-rigid ICP framework introduced in [10]. The difference is that we search for multiple two-way correspondences rather than single one-way correspondences between the source and the target. The algorithm is built around an energy minimization process similar to [11], where the energy contains a distance term and a potential energy term. Potential energy terms of [10] or [12] can be used with our multiple two-way correspondences. However, a slightly modified potential energy term allows the surface to deform more freely. This distance terms of existing methods [11, 10, 8, 9, 12, 13] can also be used with multiple two-way correspondences.

![Figure 1: The source and target surfaces.](image)

### 1.1. Contribution

This paper focuses on automatically non-rigidly registering human bone shapes and further building the statistical shape models fully automatically, providing the basis for distortion organ registration, intro-patient registration and atlas registration, which are crucial to disease analysis and treatment planning. In the field of Biomechanics, the registration step is always performed by human and is labor intensive. The registration process for each pair of human vertebrae models shown in our paper often takes 3 hours for a Biomechanics graduate student. In addition, the manual registration may also be subjective. To alleviate these problems, we develop a fully automatic method to help biomechanics researchers. The main contributions of this paper are

1. a multiple two-way correspondence search scheme,
2. a Laplacian-based potential energy, and
3. giving a way of registering similar shapes without any prior assumptions on underlying deformations, such as rigid, isometric. This registration achieves the requirements of biomechanics researchers, in addition, does not bring any subjective bias about what the deformation should be. Therefore, the method is suitable for any biomedical surface registration and the following statistical shape analysis.

Most of the existing registration methods rely on single correspondence, i.e. they assign a single point on the target surface to a point on the source surface. However, this is inadequate in many cases. For example, when there exist false single correspondences, the registration may be trapped at local minima. When the source and the target are relatively far away, multiple correspondences in both directions (source to target and target to source) can be used to facilitate the registration. This scheme serves as a heuristic for pulling the registration process out from local optima. Local geometric information (in our case, normals) is used to remove incorrect correspondences. This improves the robustness of our approach while keeping the objective function simple. A Laplacian-based potential energy is also proposed. This regularization term works better than others when incorporated with the new correspondence search scheme.

The multiple two-way correspondence search scheme is similar to coarse-to-fine correspondence search presented in [9] or [14]. In these papers, coarse alignment is firstly performed on some pre-computed features of a mesh, and then propagated to other places via diffusion [9] or local weighted combinations [14]. However, because of measurement errors and natural variability of shapes, extracted feature points may not be accurate and may not be in one-to-one correspondence. More comprehensive review of the related work, including other methods based on multiple correspondences, can be found in Section 2.
2. Related work

In this section, we briefly review the existing surface registration schemes, focusing on those based on the nonrigid ICP method. More complete review of recent results on surface registration can be found in survey paper [15].

2.1. Surface registration

In rigid registration, the space of transformations is low-dimensional. However, in nonrigid registration, the amount of allowed shape deformations is model-dependent and high dimensional [16]. When there is small distortion between the source and the target, the nearest neighbor correspondence is adopted as an initial guess. The deformation is controlled by a regularized version of this correspondence field. Andresen et al. apply an explicit convolution to the nearest neighbor displacements [8, 9] to form the initial correspondences in each iteration of the registration. In [17] and [14], thin-plate splines are used to regularize the the point movement. In [12], an implicit regularization term based on Laplacian coordinates is proposed. While it preserves the shape well, it tends to strongly limit the deformation and therefore it is not suitable for our application. In [10], a locally affine regularized point motion is used along with the nearest neighbor correspondence criterion. Mitra et al. [18] present a space-time surface registration algorithm. The input to their algorithm is a large number of scans parametrized by time, rather than a pair of surfaces.

However, when the required deformation is larger, the above-mentioned methods often yield unsatisfactory results. The current strategy of dealing with larger deformation is to add restrictions on the deformation, or on the input surfaces.

Some methods impose restrictions on input surfaces. For example, many methods [19, 20, 21, 22] rely on conformal mapping or spherical parameterizations, such as Möbius transform. The basic idea is to first map the surfaces to planar domains or spheres [23, 24, 25] and then solve the matching problem in the other domain, which is better studied. The input surfaces of these methods must have genus 0. Carrying over these techniques to higher genus surfaces would require cutting the source and target surfaces in a consistent manner, which by itself is a hard problem.

Some other methods impose restrictions on required deformations. For example, Chang and Zwicker’s method [26, 27, 28] and Huang et al.’s method [3] perform well on modeling articulated motions between surfaces.

2.2. Nonrigid ICP

The classic registration method is the Iterated Closest Point (ICP) algorithm [29, 30] (originally developed to solve the rigid registration problem). It works by iterating the following three steps:

1. Correspondence determination: select points on the source surface (source points). Pair each source point with its closest point (target point) on the target surface.
2. Error minimization: compute a rigid transformation $T$ that minimizes the mean square error between the source points (transformed with $T$) and their corresponding target points.

3. Update: apply transformation $T$ to the source surface; stop if $T$ is close to the identity transformation.

ICP is a local optimization algorithm, which generally requires close initialization (i.e. that the input source and target surfaces are close to being aligned) to converge. If initialization is close enough, the method is robust and usually converges monotonically.

Numerous improvements and extensions of the ICP algorithm have been proposed. Some of them focus on the correspondence determination stage. Examples include [17], based on fuzzy correspondences, Expectation Maximization based iterated closest point (EM-ICP) [31] and a method based on a priori knowledge [32]. By altering the error minimization step, one can develop nonrigid registration algorithms. The key is to change the optimization problem in a way that makes non-rigid transformations feasible, for example as described in [10, 17, 32]. In [33], a non-Euclidean distance which depends on a unit normal difference is used as the distance between two surfaces. This accelerates convergence, but makes the optimization problem non-quadratic, and thus increases the computational complexity.

### 2.3. Other multiple-correspondence-based schemes

Using multiple correspondences to avoid local minima is not a new idea [17, 31, 34, 35, 36, 37, 38]. The basic motivation of doing so is that matching single points is not as robust as matching clusters of points. Specifically, when there are some assumptions on motion of points, like rigidity [34, 31] or articulated motion [26, 27, 28, 39, 40, 4], matching patches or grouped points rather than single points is always the right way to go.

The main purpose of our method is to discover the variation space between a set of similar surface models with no priori information (Section 6). Therefore, we cannot impose rigidity or articulation constraint on the deformation. Our method does not require the motion to be rigid or articulated and is designed to work with surfaces rather than feature point set. When methods that are designed for registering articulated shapes are applied to input surfaces we are interested in, the resulting deformation is typically of poor quality. For example, the registration result of one pair of the vertebrae obtained by Chang and Zwicker’s method [27] contains a number of creases. Such crease-like artifacts arise because the algorithm attempts to cluster points into a small number of rigid parts. As shown in Figure 2, the joints between different parts of the mesh becomes creases.

In addition, our method does not require two shapes to be registered to have comparable sampling densities. We compare the performance with Coherent Point Drift method by Myronenko and Song [37] and Jian and Vemuri’s method [36, 38] in Section 5.

### 2.4. Biomedical and biomechanical applications

Surface mesh registration plays an increasingly important role in biomedical and biomechanical applications, e.g. [8, 24, 21, 41]. A comprehensive review related to mesh usage in medical image can be found in [42].

Andresen et al. [8] register mandible models of patients in different ages to perform the growth modeling. Huang et al. [24] register ventricular inner surfaces and hippocampal shapes for the purpose of spatiotemporal modeling.

Zeng et al. [21] register supine and prone colon surfaces to help cancer diagnosis.

Statistical shape modeling is the most important goal of registration [5, 11, 40, 4]. Gilles and Pai present a musculoskeletal registration method [40] based on surface models. The improved version of their work [4] focuses on incorporating prior information into the registration process via a plastic deformation step. In addition, their method performs well on joints between the rigid parts.

Specimen-specific finite element modeling plays a crucial role in biomechanics. To build this finite element model over a set of different shapes, registration is required. Sigal et al. [6] present both automated-wrapping-based and manual-landmarks-based surface morphing algorithms, based on spherical parametrization, to build the finite element model of rat vertebrae. The purpose of our algorithm is most similar to Sigal’s [6], which is also used for finite element modeling. However, let us stress that the human vertebrae models are genus-1 surfaces, while rat vertebrae are only genus-0 surfaces. This makes spherical parametrization based methods problematic for our application.

Recently, Lam et al. [43] present a landmark-based genus-one surface registration method via Teichmuller extremal mapping; Wen et al. [44] present a landmark constraint registration method for high-genus surfaces, such
as vestibular system. The registrations defined in these papers are both landmark-based. Our method requires no landmark picking and matching. The motivation of our method is to substitute the manual morph with an automated process in [45].

### 3. Overall procedure

In what follows, by \( S \) and \( T \) we denote the source and target surfaces. In our implementation, both \( S \) and \( T \) are triangle meshes of generally different connectivity. Our algorithm constructs a sequence of meshes \( R_0 = S, R_1, R_2, \ldots, R_n \) of identical connectivities, that tend to get closer to surface \( T \). Meshes \( R_0, R_1, R_2, \ldots, R_n \) have the same connectivity as the source mesh \( S \). In order to obtain \( R_{f+1} \) from \( R_f \), we solve an optimization problem whose objective function combines the distance term and the energy term. The distance term ensures that the surfaces represent increasingly better approximation of \( T \). The energy term attempts to preserve the quality of the mesh. The algorithm terminates when it reaches the maximum number of iterations or \( \text{dist}(R_n, T) < \epsilon \), where \( \epsilon \) is a user-specified threshold.

#### 3.1. Distance energy

In order to obtain the distance term, we first determine a set of pairs of corresponding points \((p, t_p)\), with \( p \in R_f \) and \( t_p \in T \), using the method described in Section 4. The distance term is the sum of squared distances between points in all correspondence pairs, i.e.

\[
D = \sum \| p - t_p \|^2
\]

Note that neither \( p \) nor \( t_p \) are confined to a vertex of the mesh. Using surface-to-surface distance gives better result than using vertex-to-vertex distance.

#### 3.2. Potential energy

A deformation model, that restricts the movement of mesh vertices of the deformed mesh relative to each other, is usually required in the nonrigid registration procedure to prevent the mesh quality from deteriorating. This movement control can be imposed explicitly or implicitly.

An example of an explicit scheme is described in [8] and [9]. They use geometry-constrained diffusion to convolve the deformation field. Then the convolved displacement field is used to warp the current surface to get a new surface.

In [46, 47, 13, 33], implicit deformation models are proposed based on differential geometry. In these methods, the surface is required to be parameterized before one can calculate the Gaussian curvature [46], Gaussian radial basis function [47] or unit normal [33].

These characteristics are used to define the objective function that controls the deformation. However, the requirement to parameterize the input surfaces restricts these techniques to genus-0 surface setting. For surfaces of higher genus, parameterization would require cutting the input surfaces in a consistent manner, which is hard to do.

#### 3.2.1. Combinatorial Laplacian

We choose an implicit regularization scheme, which is similar to those described in [11, 10, 12]. We use the following potential energy based on the combinatorial Laplacian operator:

\[
G(x) = \sum_i \| \Delta x^i - \Delta x^0 \|^2,
\]

where \( x^i \) is the vector obtained by concatenating coordinates of all vertices of \( R_i \), \( x^i \) represents the 3D coordinates of the \( i \)-th vertex of \( R_i \) and \( \Delta x \) is the sum of vectors running from vertex \( i \) to all of its neighbors along the mesh edges.

In practice, the deformation regularized by using this energy term is different than the deformation obtained using the edge based terms described in [11, 10, 12]:

\[
G(x) = \sum_{(i,j) \in E} \| L_{ij} x^{i+1} - L_{ij} x^0 \|^2,
\]

\( 5 \)
where $E$ consists of all pairs $(i, j)$ such that the mesh contains an edge connecting $i$-th and $j$-th vertex and $L_{ij}x = (x_j - x_i)$ is the vector running along the edge from vertex $i$ to vertex $j$.

The intuitive motivation of substituting Equation (3) with Equation (2) as the regularization of the deformation is that we found in biomedical models, saddle points in one mesh often correspond to convex points in another mesh, as shown in Figure 11. This observation is based on experiments. Such shallow saddle points appear in the model only because of the acquisition error. The traditional potential energy described in Equation (3) would attempt to preserve these saddles. From the comparisons that will be shown in following sections, one can see that our energy term allows the mesh to deform more freely. Especially when cooperating with the multiple two-way correspondence search, our energy term will achieve a better result. In what follows, we call our potential energy term (Equation (2)) the Laplacian-based energy term, and the term given in Equation (3) – the edge-based energy term.

Note that $x^T + 1$ is unknown, and $x^0$ is known since it represents the source mesh.

3.2.2. Geometry-based Laplacian

The combinatorial Laplacian operator $\Delta x$ described in Equation (2) does not take the geometry into account. In some cases, especially if the surface sampling is far from regular, it should be replaced with a geometry-aware discretization of the Laplacian. A number of such discretizations are described in [48]. In this work, we use the formulation due to Desbrun et al’s [49], based on the following formula:

$$\Delta x = \sum_{j \in N_i} \frac{\cot \alpha_{ij} + \cot \beta_{ij}}{2} (x_j - x_i),$$

where $A_i$ is the total area of all triangles incident to $x_i$, $N_i$ is the set of indices of 1-ring neighbor vertices of $x_i$, and $\alpha_{ij}$ and $\beta_{ij}$ are triangle angles shown in Figure 3.

![Diagram](image)

Figure 3: Definitions of angles $\alpha_{ij}$ and $\beta_{ij}$.

3.3. Energy minimization

The total energy to be minimized is a weighted sum of the distance energy and the potential energy. The distance energy term as well as the potential energy term are quadratic functions, therefore the objective function is quadratic:

$$E = D + \alpha G = x^T A x - 2b^T x + c,$$

where $A$, $b$, and $c$ represent its quadratic, linear and constant terms. Here, $D$ is the distance energy mentioned in Equation (1). This objective function $E$ is minimized for $x = A^{-1}b$. This is a sparse linear system that can be solved
using the standard conjugate gradient method. The potential energy term can be further explicitly expressed in terms of the Laplacian:

\[
G = \| \Delta_i (x^f - x^0) \|^2 = [\Delta_i (x^f - x^0)]^T \Delta_i (x^f - x^0) = (x^f - x^0)^T \Delta_i^T \Delta_i (x^f - x^0),
\]

where \( \Delta_i \) is the geometry-based Laplacian operator defined in Equation 4, \( x^f \) and \( x^0 \) are vertex coordinates in the current mesh and the source mesh, respectively. The quadratic and linear term contributed by \( G \) are \( \Delta_i^T \Delta_i \) and \( 2 \Delta_i^T \Delta_i x^0 \).

The parameter \( \alpha \) is adjusted adaptively during registration. For small values of \( f \), \( R_f \) is far away from \( T \). Thus, \( \alpha \) must be relatively larger to protect the mesh quality under a potentially large deformation. When the deformed surface becomes close to the target, we use a smaller value of \( \alpha \). In practice, we set

\[
\alpha = 100/1.1^f.
\]

Recall that \( f \) is the iteration number. The results are similar if 1.1 in the denominator is replaced by a similar value.

4. Correspondence search

Possible correspondence pairs are determined at the beginning of each fitting step between current surface \( R_f \) and target surface \( T \). The simplest, but often unreliable way of finding correspondences between the two surfaces is based on proximity. To increase the reliability, one can use 2-way correspondences [50, 16, 51] or incorporate local normals [33, 11, 8, 12, 52]. By “2-way”, we mean that the correspondences are searched for both from the source to the target (forward correspondence) and from the target to the source (inverse correspondence).

These improvements of the simplest proximity-based scheme are often insufficient when some parts of the source surface are far from the corresponding parts of the target surface: the correspondences may be still unreliable. Filtering correspondences based on normal vector information (i.e. disregarding them if the corresponding points’ normals significantly differ) can be an effective heuristic to remove false correspondences. However, it can potentially lead to problems shown in Figure 4(a), where some vertices in the source (blue) mesh are have no correspondences in the target (pink) mesh that pass the normal test. This leads the fitting procedure to a poor result (Figure 4(c) and (d)).

The problem is inherited from the basic ICP algorithm. In some cases, nearest correspondences are not correct correspondences. To alleviate this problem, we improve the searching procedure by using more correspondence pairs in the low confidence regions, where the source and the target meshes are far away.

In both forward and inverse correspondence search, instead of finding just one nearest vertex with similar normal information, we search for multiple possible correspondences. \( k \) possible correspondences on a mesh \( B \) for one vertex \( a \) of a mesh \( A \) with normal \( n_a \) are found as follows.

- **Step 1:** Search for \( k \) vertices of \( B \) closest to \( a \). Among these \( k \) vertices, find the one (\( b_0 \)) closest to \( a \) that has a similar normal (i.e. such that the angle between its normal and \( n_a \) is less than 90°). If no such vertex exists, \( a \) is regarded as an outlier in the forward correspondence search. Otherwise, let \( d \) be the distance between \( a \) and \( b_0 \).

- **Step 2:** Find all vertices of \( B \) whose distance to \( a \) is smaller than \( 2d \). For any such vertex \( b \) that has normal similar to \( n_a \), search the incident triangles to find the closest point \( t_a \) to \( a \). Use all resulting pairs \((a, t_a)\) as correspondence pairs.

Note that:

- This process is used both to find forward and backward correspondence pairs.
- The factor of \( 2d \) in step 2 is application dependent. For our data, choosing a value between \( 2d \) and \( 3d \) will give similar results.

Intuitively, vertices in a region on the current surface that is far away from the target have more correspondences. Since all of them contribute to the distance term, the deformation field is essentially smoothed in such areas, which
(a) overlapped view and correspondences found by just using the 2-way correspondence search and normal selection in the part where large deformations exist. The 2-way+normal based procedure fails to produce a high quality deformation in the boxed area.

(b) 2-way+Normal
(c) 20 iterations

(d) 2-step Multiple
(e) 20 iterations

Figure 4: The effect of two-step multiple correspondence search. In (b) and (e), the black lines and the red lines represent the forward correspondences and the inverse correspondences. The source and the target surfaces are shown in Figure 1.
is highly desirable since it helps to preserve the mesh quality. When two surfaces become closer to each other, the number of possible correspondences is decreasing and smoothing does not take place.

We show the effect of this scheme in Figure 4. Here, “2-way+Normal” stands for using both the 2-way one-to-one correspondence finding and the normal based correspondence trimming. Fitting results are shown in (c) and (d). “2-step multiple” stands for using the two-step multiple correspondence selecting scheme, described in this section. Results are shown in (f) and (g).

In our procedure, we use $k = \frac{50}{127} + 1$, where $f$ is the number of iterations. When two surfaces are getting closer, we can use fewer correspondences.

5. Evaluation

Here, we use the same source and target meshes shown in Figure 4 to evaluate the quality of results obtained using different methods. The source mesh is a template mesh created by biomechanical engineers, target meshes are reconstructed from CT-scans of patients.

5.1. Data description

The input data for our methods are surface models of human femurs and vertebrae. Typical source (template) and target meshes are shown in Figure 1. They are extracted from 3D images captured from CT-scans. The vertebrae surfaces

- are watertight, have no boundary;
- are of genus 1;
- are not related by deformations that can be described as articulated;
- have varying sampling rates over the space, e.g., the template mesh has over 7700 vertices while some specimens only have 4000 vertices; moreover, the surfaces typically have more samples around intrusions than flat parts;
- are similar in shape and pose;
- do not have consistent features that can be reliably determined by an automated algorithm.

Because of these characteristics, many existing methods cannot be directly used on our models. We show direct comparisons with other state of art methods on our models in section 5.

5.2. Comparison to single correspondence schemes

We compare the registration results of our method with methods incorporating single correspondence search in Figure 5.

Andresen2001: The method presented by Andresen and Nielson [9], where they use their method in mandibular registration [8]. Their method is based on displacement diffusion, which is an explicit regularization. Displacements between source and target are estimated explicitly by using nearest point search, and then a convolved displacement field is used to update the mesh. In [9], authors state that one does not have to use the crest line, which is used in [8].
and [5], a geometrical feature for the initial registration. Therefore, we here just use displacement vectors between nearest point pairs from source to target as the initial displacement field. The time-step of the diffusion stage is chosen as 0.1. For simple and models like mandible and cubes shown in their paper, their method is efficient and accurate, especially when the source and target meshes are similar. However, for complex models like vertebrae, their method fails to give a satisfiable outcome. The reason is that the directly found displacements before diffusion step are often far from reliable. Even after the diffusion step, the displacements are mostly wrong. For complex meshes, like vertebrae shown in this paper or skull models shown in [5], interactively chosen feature lines may greatly improve the registration result.

**Pauly2005**: The method presented by Pauly et al, [12], where a similar framework is used. It is also an energy minimization based scheme. The differences between their method and the proposed method include:

- They use single correspondence rather than two-way multiple correspondences.
- Their distance energy term is weighted with a confidence value, determined by bidirectional closest point search.
- Their potential energy term is area-weighted edge-based energy term.

“Bidirectional closest point search” of [12] means the following: Suppose that a point $p$ in the source mesh has a correspondence point $t_p$ in the target. One can use nearest neighbor search to determine an inverse correspondence of $t_p$ in the source mesh, named $q$. If $p$ and $q$ are close, then the correspondence between $p$ and $t_p$ is more likely to be correct. More details can be found in section 4 of [12]. From the result we can see that their method preserves the mesh well, but fails to push the source mesh to the relatively far target. In this case, single correspondence weighted with bidirectional closest point search is not enough to give us correct correspondences. As shown in Figure 6(b), the inverse correspondence (black dot) of the corresponding point (red dot) on the target is far away from the source point (blue dot). However, the false correspondence (green dot) of the point at black dot has an even closer inverse correspondence point (purple dot). In this case, the summand of the distance term that related to the false correspondence will be assigned a much higher weight.

![Figure 6: Bidirectional closest point search.](image)

We show corresponding $l_1$, $l_2$ and $l_{\text{max}}$ distances between the current mesh and the target mesh in Table 1 with different methods. $l_1$, $l_2$ and $l_{\text{max}}$ distances between the current mesh $R$ and the target mesh $T$ are defined by

\[
l_1(R, T) = \frac{1}{|R|} \int_{R, p \in R} |d(p)|,
\]
\[
l_2(R, T) = \sqrt{\frac{1}{|R|} \int_{R, p \in R} d(p)^2},
\]
\[
l_{\text{max}}(R, T) = \max_{p \in R} d(x_i),
\]
a consequence of more aggressive pursuit of the fit quality goal. The apparent distortion for our technique is slightly higher than for some other methods, which is due to different sampling rates over two surfaces. The code we use to perform the comparisons is provided by authors: http://code.google.com/p/gmmreg/

model is elongated. This phenomena is also due to different sampling rates over two surfaces. In addition, a set of control point is used to accelerate the algorithm. As shown in Figure 5.3, comparison to other multiple correspondence schemes

Comparisons with Coherent Point Drift [37]: Myronenko and Song [37] developed a point set registration method. We use vertex sets of triangular surfaces as inputs of the algorithm. The parameter of this method is set according to the example source code provided by authors: https://sites.google.com/site/myronenko/research/cpd. The method we choose is the nonrigid registration with low-rank kernel approximation. In addition, the acceleration part of using FGT to compute matrix-vector products is adopted. All results shown here require no more than 100 iterations to converge.

Figure 7: Comparisons with Coherent Point Drift [37] and its improved variant [38].

From the results shown in Figure 7, we see the current point set in coherent point drift method converges to an unsatisfactory state. Here, the source mesh has about 4,000 vertices, and the target has about 7,700 vertices. The sampling rates of surfaces are significantly different, and the vertex sets do not represent consistent features. Average residual errors of this method is also larger than those of our method (See Table 7). Other similar cases can be found in comparisons using synthetic data.

Jian2011: The point set registration method developed by Jian and Vemuri [38] also takes vertex sets of triangular surfaces as inputs. In addition, a set of control point is used to accelerate the algorithm. As shown in Figure 7, the model is elongated. This phenomena is also due to different sampling rates over two surfaces. The code we use to perform the comparisons is provided by authors: http://code.google.com/p/gmmreg/.

To see the distortions after registration, we have put stripe textures on the deformed surfaces as shown in Figure 8. The texture coordinates used here are z- (column 1), y- (column 2) and x-coordinates (column 3) of the source mesh, respectively. The apparent distortion for our technique is slightly higher than for some other methods, which is a consequence of more aggressive pursuit of the fit quality goal.

Table 1: Comparisons with other methods with different inputs.

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where $p$ denote the points on $R$, $|R|$ denotes the surface area of $R$. Here, NICPS is our method. All distances are expressed in millimeters.

Table 1 compares results obtained by fitting the template mesh to 14 vertebrae shapes used to build statistical shape model (Section 7).

Both Pauly2005 and our method are aimed to decrease the L2 distance between source and target meshes. One can observe that our method achieves better performance: the average L2 distance is approximately 34% of Pauly2005’s.

5.3. Comparison to other multiple correspondence schemes

Coherent Point Drift: Myronenko and Song [37] developed a point set registration method. We use vertex sets of triangular surfaces as inputs of the algorithm. The parameter of this method is set according to the example source code provided by authors: https://sites.google.com/site/myronenko/research/cpd. The method we choose is the nonrigid registration with low-rank kernel approximation. In addition, the acceleration part of using FGT to compute matrix-vector products is adopted. All results shown here require no more than 100 iterations to converge.
Figure 8: Results shown with textures.
Figure 9: Results shown with textures.
5.4. Edge-based energy vs Laplacian-based energy

If we incorporate our two-way multiple correspondence search scheme into Pauly2005 to substitute the bidirectional closest point search scheme, we will get results shown in Figure 10a, where an uncomfortable distortion in the deformed mesh is visually apparent.

![Figure 10: Results comparison between edge-based and Laplacian-based potential energy with two-multiple correspondence. The pink surfaces are deformed models.](image)

However, if we use Laplacian-based potential energy rather than edge-based potential energy, the result looks much better.

The difference is due to the fact that the correspondences of saddle points in the source mesh are not saddle in the target. See Figure 11. Edge-based potential energy will try to preserve this saddle, however, the Laplacian-based will not. Generally, Laplacian-based potential energy allows the mesh to deform more freely.

![Figure 11: A saddle point in the source mesh.](image)

5.5. Feature preservation

24 feature points are manually picked by biomechanical engineers on both source and target meshes, as shown in Figure 18a and c.

The feature points identified on each specimen mesh were chosen in correspondence with anatomical landmarks (Figure 17). These included points at the following locations: the most superior and posterior margin of the spinous
Table 2: Distances between landmarks before and after registration using different methods.

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Average: 5.76564 4.25529 4.0401 4.1083 7.10477 3.38939

From the results, we observe that most features are fitted to corresponding ones on target model except feature point 0 and feature point 21. See Figure 18b and d. Distances between corresponding landmarks are shown in Table 2. Our method achieves the shortest average distance.

5.6. Registering genus-0 models

All experiments designed above are performed on Human Vertebrae data, which are generally genus-0 meshes. Here, we show a registration on Human Femur data. This experiment illustrates that NICPs is also suitable for genus-0 meshes.

![Figure 12: First Source (blue) femur model to Target (pink) model yielding Fitted model (purple).](image)

As shown in Figure 12, two input femur models have different length and shapes. They are twisted differently. NICPs successfully fit the source mesh to the target mesh.

6. Application: statistical shape analysis

We use our registration method to analyze the shape variation of a family of femur models and a family of vertebrae models. Similar approaches can be found in [53, 5]. We applied our procedure to compute compatible triangulations of a set of 14 vertebrae and 14 femur models.
6.1. Femur models

We choose one of the 14 input femur models $P_0$ as the base mesh. Then, we deform the base mesh to each of 14 meshes $P_i$ ($i = 0, 1, ..., 13$) obtaining a new sequence of models $P'_i$ ($i = 0, 1, ..., 13$) (note that $P'_0 = P_0$). $P'_i$ is geometrically close to $P_i$ and all meshes $P'_i$ have the same connectivity as the base mesh $P_0$. The average shape $P_{\text{mean}}$ can be computed by simply averaging the vertex coordinates over all of the deformed models (Figure 13).

For each deformed model $P'_i$ ($i = 1, 2, ..., 13$), we define the 3n-dimensional shape vector $S_i = P'_i - P_{\text{mean}}$, where $n$ is the number of vertices of the base mesh $P_0$. This set of shape vectors $S_i$ spans a space of the input shapes. We perform principal component analysis (PCA) on the resulting set of shape vectors to find predominate shape variations (eigenshapes).

The first three principal components (PC) of variations captured approximately 93.9% (69.8% for the first PC, 18.5% for the second PC and 5.6% for the third PC) of the variance of the data. Their effects to the shape changing are illustrated in Figure 13.

![1st PC](image1)
![2nd PC](image2)
![3rd PC](image3)

Figure 13: First three shape variation components. The blue shape is the average model $P_{\text{mean}}$, the pink shapes are three eigen models which are created by adding three PCs to $P_{\text{mean}}$, respectively.

One can see that

- the first PC controls the length;
- the second PC controls the twist;
- the third PC controls the bend.

6.2. Vertebrae models

For the vertebrae models, we fit a template model to 14 vertebrae models (Figure 14) and use the same procedure to construct PCA models. In this case, the template mesh is a highly regular mesh constructed manually [45]. We proceed in the same way as in Section 6.1.

![Vertebrae models](image4)

Figure 14: Vertebrae models before registration. (a) shows the template model. (b) and (c) show overlapping views of 14 models.
The variation of vertebraes is more complex than that of femurs. In particular, first three principal components of variations capture symmetric aspects of the shapes, as shown in Figure 15.

![Figure 15](image-url) Figure 15: First five shape variation components. The blue shape is the average model $P_{\text{mean}}$, the pink shapes are three eigen models which are created by adding three PCs to $P_{\text{mean}}$, respectively.

The same task can also be accomplished by using an interactive fitting methods [45]. The results are shown in Figure 16. Note that the eigenshapes are similar, although not the same (since they are based on different correspondence information). Our fitting procedure requires a minimum amount of user input (just to roughly align the input shapes) and therefore make shape analysis less dependent on human labor.

7. Conclusions and limitations

We propose a two-way multiple correspondence search scheme for nonrigid surface registration. To take full advantage of this scheme, one should adopt a relatively looser potential energy term, for example, the Laplacian-based potential energy term. The scheme does not rely on parameterizations and can be applied to families of complex models with relatively high shape variabilities, such as vertebrae.

Intuitively, multiple correspondences act like a smoothing filter, with the amount of smoothing proportional to local distance between the source and the target surface. All correspondences for a vertex of the source mesh are essentially “averaged” by the potential energy term. This is exactly what is desired in nonrigid shape registrations: ideally, if the distance between the source and the target model is large, one should focus on large scale geometric features when computing the deformation. Smaller scale features should influence the deformation on later stages. This observation has been exploited in coarse-to-fine multigrid deformation such as [4].

The proposed method is limited in several aspects:
Figure 16: First three shape variation components got by using an interactive fitting method. The blue shape is the average model $P_{\text{mean}}$, the pink shapes are three eigen models which are created by adding three PCs to $P_{\text{mean}}$, respectively.

Figure 17: 24 feature points shown on the source mesh with different views.

Figure 18: Feature points shown on source, target and fitted mesh.

- The computational time is longer than traditional methods. In the example shown in the comparison section, average running times for 10 iterations of Pauly2005’s method, Andresen2001’s method and ours are 153, 227 and 379 seconds. The multiple correspondence scheme is approximately one and half times slower than only using one-to-one correspondences.

- It is not clear about how many correspondences are suitable in each step of the registration. The problem is similar to the problem of coarse-to-fine strategy used by Gilles et al.[4], where the cluster size is manually decreased.

- The current version of multiple correspondence search algorithm cannot distinguish similar patterns. For example, our method may deform two horse legs into one or swap them.

Besides, there are a number of parameters that need be chosen for different types of inputs. Among them, the weight and form of potential energy function are crucial. Intuitively, the more correspondences we use, a more
flexible potential energy is required. In addition, correspondence trimming method can be further improved. Besides trimming correspondences according to normal differences, more sophisticated criteria, such as multi-level curvatures used in Huang et al’s method [3] and spin images used by Chang and Zwicker [27] can be used.

The main goal of this work is to substitute the manual fitting by biomechanics engineers [45], and then perform a more objective statistical shape analysis. The shape variation information is then used to build specimen-specific Finite Element (FE) models [45, 6]. Since the vertebrae models are manifolds, using Principle Geodesic Analysis (PGA) [7] instead of PCA may yield more meaningful results.

Acknowledgment

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